

# Leptogenesis in $SO(10)$ models with a left-right symmetric seesaw mechanism

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**Abstract.** We study leptogenesis in supersymmetric  $SO(10)$  models with a left-right symmetric seesaw mechanism, including flavour effects and the contribution of the next-to-lightest right-handed neutrino. Assuming  $M_D = M_u$  and hierarchical light neutrino masses, we find that successful leptogenesis is possible for 4 out of the 8 right-handed neutrino mass spectra that are compatible with the observed neutrino data. An accurate description of charged fermion masses appears to be an important ingredient in the analysis.

**PACS.** 12.10.Dm Unified theories and models of strong and electroweak interactions – 14.60.St Non-standard-model neutrinos, right-handed neutrinos, etc.

## 1 Introduction

Testing the seesaw mechanism [1] is almost certainly an hopeless goal, except for specific low-energy realizations. The main reasons we have to believe in it are its elegance and the fact that it fits so nicely into  $SO(10)$  unification. This motivates us to investigate its observable implications, such as leptogenesis [2] and, in supersymmetric theories, lepton flavour violation.

So far most studies of leptogenesis have been done in the framework of the type I (heavy right-handed neutrino exchange) seesaw mechanism, or assumed dominance of either the type I or the type II (heavy scalar  $SU(2)_L$  triplet exchange) seesaw mechanism. It is interesting, though, to investigate whether the generic situation where both contributions are comparable in size can lead to qualitatively different results. A further motivation to do so comes from the well-known fact that successful leptogenesis is difficult to achieve in  $SO(10)$  models with a type I seesaw mechanism, which generally<sup>1</sup> present a very hierarchical right-handed neutrino mass spectrum, with  $M_1$  lying below the Davidson-Ibarra bound [3].

In this talk, we present results on leptogenesis in  $SO(10)$  models with a left-right symmetric seesaw mechanism. Details can be found in Refs. [4, 5] (for related work, see Refs. [6, 7]).

## 2 Right-handed neutrino spectra in the left-right symmetric seesaw mechanism

### 2.1 The left-right symmetric seesaw mechanism

In left-right symmetric extensions of the Standard Model, the light neutrino mass matrix is often given by the following formula [8]:

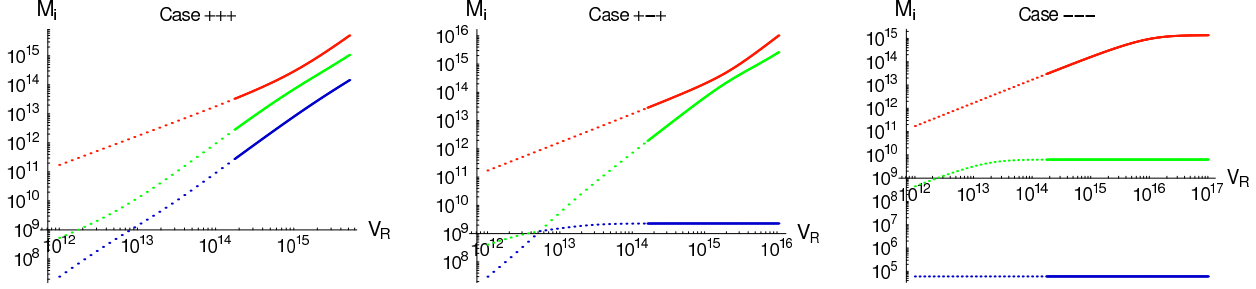
$$M_\nu = f v_L - \frac{v^2}{v_R} Y_\nu^T f^{-1} Y_\nu. \quad (1)$$

In Eq. (1),  $v_R$  is the scale of  $B - L$  breaking,  $v$  is the electroweak scale, and  $v_L \sim v^2 v_R / M_{\Delta_L}^2$  is the vev of the heavy  $SU(2)_L$  triplet. A discrete left-right symmetry ensures that a single symmetric matrix  $f$  determines both the couplings of the  $SU(2)_L$  triplet to lepton doublets, to which the type II contribution (first term) is proportional, and the right-handed neutrino mass matrix  $M_R = f v_R$ , which enters the type I contribution (second term). The discrete symmetry also constrains the Dirac coupling matrix  $Y_\nu$  to be symmetric.

In order to study leptogenesis, the knowledge of the masses and couplings of the right-handed neutrinos and of the  $SU(2)_L$  triplet is needed. Therefore, in a theory which predicts the Dirac matrix  $Y_\nu$ , one must solve Eq. (1) for the  $f_{ij}$  couplings, assuming a given pattern for the light neutrino masses and mixings. In Ref. [9], it was shown that this “reconstruction” problem has exactly  $2^n$  solutions for  $n$  families, and explicit expressions for the  $f_{ij}$ ’s were provided up to  $n = 3$ . Here we use the alternative reconstruction procedure proposed in Ref. [4].

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<sup>1</sup> This might not be the case in models where the relation  $M_D = M_u$  receives large corrections from Yukawa couplings involving a **126** or **120** Higgs representation, or from non-renormalizable interactions.



**Fig. 1.** Right-handed neutrino masses as a function of  $v_R$  (in GeV) for solutions  $(+, +, +)$  (left),  $(+, -, +)$  (middle) and  $(-, -, -)$  (right panel). Inputs: hierarchical light neutrino masses with  $m_1 = 10^{-3}$  eV,  $\sin^2 \theta_{13} = 0.009$ ,  $\beta/\alpha = 0.1$  and no CP violation beyond the CKM phase. The range of variation of  $v_R$  is restricted from above by the requirement that  $f_3 \leq 1$ . Dotted lines indicate a fine-tuning greater than 10% in the  $(3, 3)$  entry of the light neutrino mass matrix.

## 2.2 Reconstruction procedure

In order to solve Eq. (1), we first rewrite it as

$$Z = \alpha X - \beta X^{-1}, \quad (2)$$

with  $\alpha \equiv v_L$ ,  $\beta \equiv v^2/v_R$  and

$$Z \equiv N_\nu^{-1} M_\nu (N_\nu^{-1})^T, \quad X \equiv N_\nu^{-1} f (N_\nu^{-1})^T, \quad (3)$$

where  $N_\nu$  is a matrix such that  $Y_\nu = N_\nu N_\nu^T$ , and  $Y_\nu$  is assumed to be invertible. Being complex and symmetric,  $Z$  can be diagonalized by a complex orthogonal matrix if its eigenvalues (i.e. the roots of the characteristic polynomial  $\det(Z - z\mathbf{1}) = 0$ ) are all distinct:

$$Z = O_Z \text{Diag}(z_1, z_2, z_3) O_Z^T, \quad O_Z O_Z^T = \mathbf{1}. \quad (4)$$

Then, upon an  $O_Z$  transformation, Eq. (2) reduces to 3 independent quadratic equations for the eigenvalues of  $X$ ,  $z_i = \alpha x_i - \beta x_i^{-1}$ . For a given choice of  $(x_1, x_2, x_3)$ , the solution of Eq. (1) is given by:

$$f = N_\nu O_Z \text{Diag}(x_1, x_2, x_3) O_Z^T N_\nu^T. \quad (5)$$

The right-handed neutrino masses  $M_i = f_i v_R$  are obtained by diagonalizing  $f$  with a unitary matrix  $U_f$ , and the couplings of the right-handed neutrino mass eigenstates are given by  $Y \equiv U_f^\dagger Y_\nu$ .

Since each equation  $z_i = \alpha x_i - \beta x_i^{-1}$  has two solutions  $x_i^-$  and  $x_i^+$ , there are 8 different solutions for the matrix  $f$ , which we label in the following way:  $(+, +, +)$  refers to the solution  $(x_1^+, x_2^+, x_3^+)$ ,  $(+, +, -)$  to the solution  $(x_1^+, x_2^+, x_3^-)$ , and so on. It is convenient to define  $x_i^-$  and  $x_i^+$  such that, in the  $4\alpha\beta \ll |z_i|^2$  limit:

$$x_i^- \simeq -\frac{\beta}{z_i}, \quad x_i^+ \simeq \frac{z_i}{\alpha}. \quad (6)$$

With this definition, the large  $v_R$  limit ( $4\alpha\beta \ll |z_1|^2$ ) of solutions  $(-, -, -)$  and  $(+, +, +)$  corresponds to the “pure” type I and type II cases, respectively:

$$f(-, -, -) \xrightarrow{4\alpha\beta \ll |z_1|^2} -\frac{v^2}{v_R} Y_\nu M_\nu^{-1} Y_\nu, \quad (7)$$

$$f(+, +, +) \xrightarrow{4\alpha\beta \ll |z_1|^2} \frac{M_\nu}{v_L}. \quad (8)$$

The remaining 6 solutions correspond to mixed cases where the light neutrino mass matrix receives significant contributions from both types of seesaw mechanisms. In the opposite, small  $v_R$  limit ( $|z_3|^2 \ll 4\alpha\beta$ ), one has  $x_i^\pm \simeq \pm \text{sign}(\text{Re}(z_i)) \sqrt{\beta/\alpha}$ , which indicates a partial cancellation between the type I and type II contributions to light neutrino masses.

## 2.3 Application to $SO(10)$ models

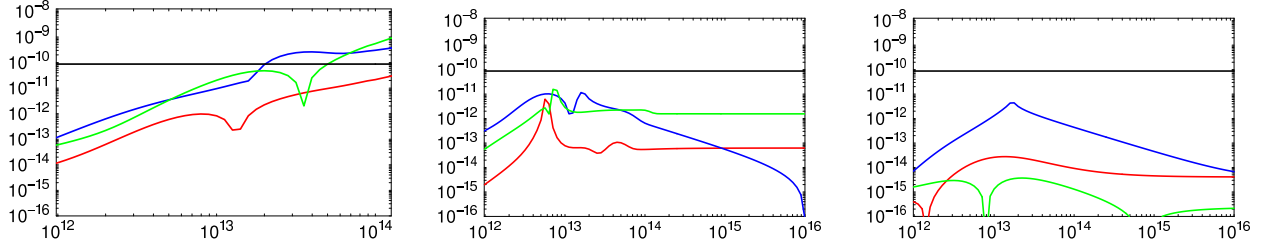
Let us now apply the reconstruction procedure to supersymmetric  $SO(10)$  models with two **10**s, a **54** and a  **$\overline{126}$**  representations in the Higgs sector. The two **10**s generate the charged fermion masses, leading to the well-known relations:

$$M_u = M_D (\equiv Y_\nu v_u), \quad M_d = M_e. \quad (9)$$

The **54** and the  **$\overline{126}$**  contain the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representations needed for the left-right symmetric seesaw mechanism. In particular, the  $SU(2)_L$  triplet as well as the  $SU(2)_R$  triplet whose vev  $v_R$  breaks  $B-L$  are components of the  **$\overline{126}$** . The equality  $f_L = f_R$  and the symmetry of  $Y_\nu$  are ensured by  $SO(10)$  gauge symmetry.

Then, for a given choice of the light neutrino mass parameters and of the high energy phases contained in  $M_u$ , the matrix  $Z$  is known<sup>2</sup> and  $f$  can be reconstructed as a function of the  $B-L$  breaking scale  $v_R$  and of  $\beta/\alpha$ . Perturbativity of the  $f_{ij}$  couplings constrains  $\beta/\alpha \leq \mathcal{O}(1)$  and restricts the range of  $v_R$  from above. In Fig. 1, we show the right-handed neutrino mass spectrum of three representative solutions as a function of  $v_R$  for a hierarchical light neutrino mass spectrum. The 4 solutions with  $x_3 = x_3^-$  are characterized by a constant value of the lightest right-handed neutrino mass,  $M_1 \approx 6 \times 10^4$  GeV; the 2 solutions with  $x_3 = x_3^+$  and  $x_2 = x_2^-$  by  $M_1 \approx 2 \times 10^9$  GeV; and the 2 solutions with  $x_3 = x_3^+$  and  $x_2 = x_2^+$  by a rising  $M_1$ .

<sup>2</sup> The implicit additional inputs are  $\tan \beta$  (we choose  $\tan \beta = 10$ ) and the values of the up quark masses and of the CKM matrix at the seesaw scale.



**Fig. 2.**  $Y_B$  as a function of  $v_R$  (in GeV) for solutions  $(+, +, +)$  (left),  $(+, -, +)$  (middle) and  $(-, -, -)$  (right panel). Inputs: hierarchical light neutrino mass spectrum with  $m_1 = 10^{-3}$  eV,  $\sin^2 \theta_{13} = 0.009$  and  $\delta_{PMNS} = 0$ ;  $\beta/\alpha = 0.1$ ; three different choices for the Majorana and high-energy phases (blue:  $\Phi_2^u = \pi/4$ ; green:  $\Phi_2^d = \pi/4$ ; red: no CP violation beyond the CKM phase); vanishing initial abundance for  $N_1$  and  $N_2$ .

### 3 Implications for leptogenesis

Since  $M_{\Delta_L} \sim (\beta/\alpha)v_R$  and  $M_1 \ll v_R$  in all solutions, one can safely assume that the  $SU(2)_L$  triplet is heavier than the lightest right-handed neutrino. Then the dominant contribution to leptogenesis comes from out-of-equilibrium decays of  $N_1$  (in some cases to be discussed below, the next-to-lightest neutrino  $N_2$  will also be relevant). The CP asymmetry in  $N_1$  decays,  $\epsilon_{N_1} \equiv [\Gamma(N_1 \rightarrow lH) - \Gamma(N_1 \rightarrow \bar{l}H^*)] / [\Gamma(N_1 \rightarrow lH) + \Gamma(N_1 \rightarrow \bar{l}H^*)]$ , receives two contributions: the standard type I contribution  $\epsilon_{N_1}^I$  [2, 10], and an additional contribution  $\epsilon_{N_1}^{II}$  from a vertex diagram containing a virtual triplet [11, 12]:

$$\epsilon_{N_1}^I = \frac{1}{8\pi} \sum_k \frac{\text{Im}[(YY^\dagger)_{1k}]^2}{(YY^\dagger)_{11}} f(x_k), \quad (10)$$

$$\epsilon_{N_1}^{II} = \frac{3}{8\pi} \sum_{k,l} \frac{\text{Im}[Y_{1k}Y_{1l}f_{kl}^*v_L^*]}{(YY^\dagger)_{11}} \frac{M_1}{v_u^2} g(x_\Delta), \quad (11)$$

where  $f(x) = -\sqrt{x}[2/(x-1) + \ln(1+1/x)]$ ,  $g(x) = x \ln(1+1/x)$ ,  $x_k \equiv M_k^2/M_1^2$ ,  $x_\Delta = M_{\Delta_L}^2/M_1^2$ , and  $Y \equiv U_f^\dagger Y_\nu$ . The final baryon asymmetry is given by:

$$Y_B \equiv \frac{n_B}{s} = -1.48 \times 10^{-3} \eta \epsilon_{N_1}, \quad (12)$$

where  $\eta$  is an efficiency factor to be determined by integrating the Boltzmann equations. For leptogenesis to be successful, Eq. (12) should reproduce the observed baryon-to-entropy ratio  $Y_B^{obs.} = (8.7 \pm 0.3) \times 10^{-11}$  [13].

The behaviour of the different solutions can be anticipated from the observation of the mass spectra in Fig. 1 [4]. Indeed, successful leptogenesis requires  $|\epsilon_{N_1}| \geq \mathcal{O}(10^{-7})$ , while for  $M_1 \ll M_2, M_{\Delta_L}$  Eqs. (10) and (11) yield the upper bound [12]:

$$|\epsilon_{N_1}| \leq 2 \times 10^{-7} \left( \frac{M_1}{10^9 \text{ GeV}} \right) \left( \frac{m_{max}}{0.05 \text{ eV}} \right). \quad (13)$$

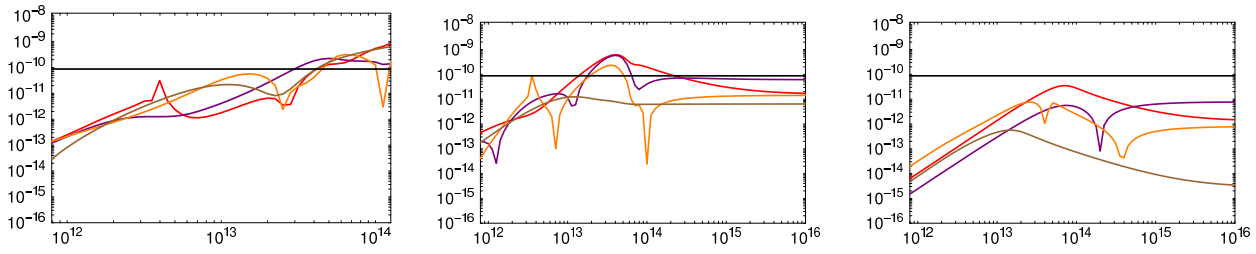
Thus, the 4 solutions with  $x_3 = x_3^-$  will fail to generate the observed baryon asymmetry from  $N_1$  decays, a conclusion that generalizes a well-known fact in the type I case. However,  $N_2$  decays can do the job if they generate a large asymmetry in a lepton flavour that is

only mildly washed out by  $N_1$  decays and inverse decays [14]. The 2 solutions with  $x_3 = x_3^+$  and  $x_2 = x_2^+$  have a rising  $M_1$  and should be able to reproduce the observed asymmetry, as in the pure type II case. Finally, the situation is less conclusive for the 2 solutions with  $x_3 = x_3^-$  and  $x_2 = x_2^-$ , for which flavour effects and the contribution of  $N_2$  could be decisive.

It is clear from the above discussion that a careful study of leptogenesis requires the inclusion of the next-to-lightest right-handed neutrino and of flavour effects [15]. As is well known in the type I case, flavour effects can significantly affect the final baryon asymmetry if there is a hierarchy between the washout parameters for different lepton flavours [16]. We performed such an analysis in Ref. [5], and present our results here. Fig. 2 shows the final baryon asymmetry  $Y_B$  as a function of  $v_R$  for solutions  $(+, +, +)$ ,  $(+, -, +)$  and  $(-, -, -)$ . Not surprisingly, the  $(+, +, +)$  solution leads to successful leptogenesis; however there is a tension with the upper bound on the reheating temperature from gravitino overproduction [17] above  $v_R \approx 3 \times 10^{13}$  GeV, where  $M_1 > 10^{10}$  GeV. By contrast, the solutions  $(+, -, +)$  and  $(-, -, -)$  fail to reproduce the observed baryon asymmetry<sup>3</sup>. In the  $(-, -, -)$  case, flavour effects prevent an exponential washout of the  $B - L$  asymmetry generated in  $N_2$  decays ( $N_1$  decays alone would give  $Y_B \sim (10^{-17} - 10^{-15})$ ), but this is not sufficient for “ $N_2$  leptogenesis” to work.

However, this is not the whole story, since the above results were obtained assuming the  $SO(10)$  mass relation  $M_d = M_e$ , which is in gross conflict with experimental data. Corrections to this formula, e.g. from non-renormalizable operators of the form  $\mathbf{16}_i \mathbf{16}_j \mathbf{10}_d \mathbf{45}$ , will modify the reconstructed  $f_{ij}$ ’s by introducing a mismatch  $U_m$  between the bases of charged lepton and down quark mass eigenstates. Fig. 3 shows how the final baryon asymmetry is modified when the effect of  $U_m$  is taken into account. We can see that several choices for  $U_m$  (the measured charged lepton and down quark masses do not fix all parameters in  $U_m$ ) lead to successful leptogenesis in the  $(+, -, +)$  case, but not in the  $(-, -, -)$  case. There is some tension between successful leptogenesis and gravitino overpro-

<sup>3</sup> In Ref. [6], a different conclusion has been obtained for the solution  $(+, -, +)$  in the case of an inverted light neutrino mass hierarchy.



**Fig. 3.** Same as Fig. 2, but with corrections to the relation  $M_d = M_e$  from the non-renormalizable operators **16<sub>16</sub>****10<sub>d</sub>****45**, keeping the relation  $M_D = M_u$ . Four different choices of the matrix  $U_m$  and of the CP-violating phases.

duction in the  $(+, -, +)$  solution but, exactly as in the  $(+, +, +)$  solution, the observed asymmetry is generated over a significant portion of the parameter space with  $M_1 < 10^{10}$  GeV.

## 4 Conclusions

We have studied leptogenesis in supersymmetric  $SO(10)$  models with a left-right symmetric seesaw mechanism, including flavour effects and the contribution of the next-to-lightest right-handed neutrino. Assuming the relation  $M_D = M_u$  and a hierarchical light neutrino mass spectrum, we found that the “type II-like” solutions  $(+, +, +)$  and  $(-, +, +)$ , as well as the solutions  $(+, -, +)$  and  $(-, -, +)$ , can lead to successful leptogenesis. An accurate description of charged fermion masses was a crucial ingredient in the analysis. By contrast, the solution  $(-, -, -)$  fails to generate the observed baryon asymmetry from  $N_2$  decays, and a similar conclusion holds for the 3 other solutions with  $x_3 = x_3^-$  if one requires  $M_1 < 10^{10}$  GeV.

Some comments about the generality of our results are in order: (i) Although the above results were obtained for  $M_D = M_u$ , the same qualitative behaviour of the 8 solutions is expected for a more generic hierarchical Dirac matrix. Of course, whether leptogenesis is successful or not in a given solution can only be decided on a model-by-model basis; (ii) At the quantitative level, different input parameters (other than the various phases and  $U_m$ ) can significantly affect the results presented in Figs. 1 to 3. This is most notably the case of the light neutrino mass parameters:  $\theta_{13}$ ,  $m_1$  and the type of the mass hierarchy (see Ref. [5] for details). Also, corrections to the relation  $M_D = M_u$  could have a significant impact, since e.g. both  $M_1$  in the  $(+, -, +)$  solution and  $M_2$  in the  $(-, -, -)$  solution are proportional to  $y_2^2 v_u^2 / m_3$ .

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